2D Modeling of a Passive Compliant Gripper

Spring Term 2013
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Abstract

The main objective of this thesis is to propose a 2D model of a passive compliant gripper which can be scaled and therefore is able to pick and place different shaped objects. The motivation of such a gripper originates from the field of robotic body extension where self-reconfigurable robots are considered which can adaptively change their structures.

The approach presented in this thesis used a lumped-parameter method in which the flexible gripper is discretized into a set of rigid bodies connected by revolute joints with torsional spring-damper systems. The interaction between the gripper and the object is represented by a soft contact model where the normal force is determined by a spring-damper system. Frictional forces are also included. The model of the gripper and the interaction were implemented in MATLAB SimMechanics. The simulation of the model based on a real gripper was validated by comparing the angle of deflection for two different shaped objects during lifting the object. The relative error of this angle between model and real world experiment is approximately 2%. The grippers are made of Hot Melt Adhesives (HMA) and were manufactured by a multi degree of freedom robot arm consisting of a fully automated glue supply mechanism.
## Symbols

**Symbols**

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>$M$</td>
<td>Bending moment</td>
<td>[Nm]</td>
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<tr>
<td>$E$</td>
<td>Young’s modulus</td>
<td>[MPa]</td>
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<td>$I$</td>
<td>Second moment of area</td>
<td>[m$^4$]</td>
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<td>$F_N$</td>
<td>Normal contact force</td>
<td>[N]</td>
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<td>$\mu_s$</td>
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<td>$\mu_k$</td>
<td>Coefficient of kinetic friction</td>
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### Indices

<table>
<thead>
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<tr>
<td>$W$</td>
<td>World Reference Frame</td>
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### Acronyms and Abbreviations

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<td>ETH</td>
<td>Eidgenössische Technische Hochschule</td>
</tr>
<tr>
<td>BIRL</td>
<td>Bio-Inspired Robotics Lab</td>
</tr>
<tr>
<td>HMA</td>
<td>Hot Melt Adhesives</td>
</tr>
<tr>
<td>GBE</td>
<td>Generalized Beam Element</td>
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Chapter 1

Introduction

1.1 Motivation

Picking and placing objects is a crucial task in robotics. These robots take a product from one location to another. The automation speeds up the process since for a human the required repetitive motion over a long duration results in possible fatigue issues. Moreover, the robots are more accurate. Such tasks have been used extensively, e.g. in auto industry. In contrast, the involvement of robotic manipulation in food industry is relatively little since the food products are highly variable in shape, sizes and structures and therefore are not easy to handle for manipulators [8]. Hence, the design of an universal gripper which is able to pick-and-place various objects of different shapes and surfaces is a challenging task.

Manipulation of objects may be divided into two fields, i.e. the task of restraining objects (fixturing) and the task of manipulating objects with fingers (dexterous manipulation) [7]. The grasping itself can be divided into two types: fingertip grasps and enveloping grasps [5]. Generally, fingertip grasps can apply an arbitrary force onto the object, since the force can be actively controlled.

Sophisticated multi-fingered hands use a lot of actuators, e.g. the hand built by the Utah-MIT is designed with 32 actuators [9]. This leads to difficulties in robot control and the actuators and sensor are generally also the highest cost components. Thus, there are number of efforts to focus on reduced complex mechanisms such as passive grippers. Sakai et al. [3] developed a pick-and-place mechanism without any sensor and actuators based on passive force/form closure principle. The mechanism consists of rigid bodies which gain mobility by hinges.

1.2 Passive Compliant Gripper

A passive gripper is from a control point of view an underactuated system. The purpose of underactuation is to reduce the amount of actuators and therefore drive the opening and closing motion of the gripper by a single actuator. On the other hand, compliance implies that the gripper is made of a flexible structure. Petković et al. [6] developed a passive compliant gripper made by press-curving from silicone. The gripper is depicted in Figure 1.1. The system is driven by a single actuator. With iterative FEM optimization and optimal criteria method using mathematical programming an optimal shape of the gripper was obtained. The optimization was
performed for two target functions: accommodation to concave and convex shapes of grasping objects.

![Figure 1.1: Gripper from Petković et al. [6]. Left: initial position. Right: grasping different shaped objects.](image)

### 1.3 Robotic Body Extension

The capability of extending body structures is one of the most significant challenges in the robotics research [1]. The idea of a self-reconfigurable robot which is able to adaptively change its structure has been partially explored. A robot that is able to extend its body structures and integrate them into its own body can accomplish new tasks which could not have been achieved otherwise. A new body structure that can be integrated is, for example, a passive compliant gripper. The robot can pick and place objects with this new manufactured tool.

#### 1.3.1 HMA Gripper

The grippers that are currently used at the Bio-Inspired Robotics Lab (BIRL) are made of Hot Melt Adhesives (HMA) and are manufactured by a multi degree of freedom robot arm consisting of a fully automated glue supply mechanism. HMA sticks are transferred from solid to liquid state inside the glue supply mechanism by heating resistors. Once in liquid state, the HMA is applied through a nozzle on the desired position. The glue supply mechanism follows a predefined trajectory forming the structure layer by layer. An example is shown in Figure 1.2. This geometric type of gripper is later modeled in Chapter 3.

![Figure 1.2: HMA Gripper manufactured by a multi degree of freedom robot.](image)

#### 1.3.2 Functional Principle

The functional principle of picking up an object is conceptually illustrated in Figure 1.3. The gripper approaches an object which is assumed to be rigid for the modeling (a). When the object and the gripper are in contact (b) the left and right beam
are bent outwards (c) due to the contact forces (discussed in detail in Section 4.3). Pushing the gripper further, the beams start to envelop the object (d). Eventually, the gripper is able to pick up the object as a result of force/form closure (e). Friction forces are central for this enveloping grasp and therefore predicting the passive force closure is crucial to robotic grasping [5].

Placing the object may be a difficult task for passive compliant grippers. E.g., in micro-assembly of microparts release is obtained by using a specific imprint on the substrate (clip, lock) [10]. In our case, placing the object is achieved by pushing the gripper further towards the ground. The left and right beam are bent further outwards and therefore the object is released. At that point, the gripper can be moved sideways out of range of the object and execute another task.

Figure 1.3: Schematic of picking up mechanism.
Chapter 2

Modeling Methods

2.1 Lumped-Parameter Method

A lumped-parameter method was used for the kinematic model of the gripper. The HMA gripper is a flexible body, i.e., a continuous medium. The lumped-parameter method approximates a flexible body as a set of rigid bodies coupled with springs and dampers [2]. Thus, this can be implemented by a chain of alternating bodies and joints in SimMechanics. The spring stiffness coefficients and damping coefficients are functions of the material properties and the geometry of the flexible member under consideration. The method is best suited to model beam-like geometries/linear geometries, in which each fundamental flexible element is coupled to two others in a simple chain. This method could also be extended to more complicated body geometries, although other approaches may be more suitable such as FEM (see Section 2.2.2).

2.1.1 Theory

In this section, the lumped-parameter method is illustrated by an example of a continuous uniform beam (see Figure 2.1). The cantilever beam of length $L$ is clamped on the left side and has a certain Young’s modulus $E$ and second moment of area $I$. The beam is discretized into $n$ identical generalized beam elements (GBEs). Each GBE has length $l = L/n$ and mass $m = M/n$ and consists of a joint-body-joint combination. On the joint acts a spring-damper system whose properties are determined by the material properties. Note that the GBEs do not necessarily have to be identical. The material properties may vary along the beam which result in different GBEs properties. Moreover, the length of each GBE may also be chosen differently, for example, a more detailed approximation to the shape of a beam portion.

2.1.2 Pure Bending

The above theory is specialized to the case of pure bending. In that case, the best model is to be expected by having a revolute joint with a torsional spring-damper system. The bending moment $M_i$ at the $i$-th GBE of length $l_i$ is given by:

$$M_i = k_i \cdot \Delta \alpha_i$$  \hspace{1cm} (2.1)  

where $k_i$ is the torsional spring stiffness at node $i$ and $\Delta \alpha_i$ is the relative angle between the $i$-th and $(i-1)$-th GBE, depicted in Figure 2.2. The torsional spring
stiffness $k_i$ can be obtained by an approximation of the equation for the transverse deflection of the beam from Euler-Bernoulli beam theory [2, 4]:

$$k_i = \frac{E_i I_i}{l_i}$$

(2.2)

where $E_i$ is the Young’s modulus of the $i$-th GBE and $I_i$ is the second moment of area at node $i$.

For a uniform beam and identical GBEs the torsional spring stiffness equates to $k = \frac{EI}{L}$.

Figure 2.2: Illustration of the $i$-th GBE.

### 2.1.3 Static Deflection of a HMA Cantilever Beam

In this section, the static deflection $d$ of an elliptical uniform HMA cantilever beam of length $L$ and with point end load $F$ is analyzed, as shown in Figure 2.3. The Young’s modulus $E$ of HMA is approximately given by 8.9MPa [1]. The relationship between the deflection $d$ and applied force $F$ is

$$d = \frac{FL^3}{3EI}.$$ 

(2.3)

Following parameters were used to compute the static deflection: $L = 30$mm, $I = 22$mm$^4$ and $F = 0.05$N which gives a static deflection of $d = 2.30$mm.

The cantilever beam was also modeled with the above mentioned lumped-parameter method in SimMechanics. The beam is represented by different numbers of GBEs in order to get an idea of the relationship between number of GBEs and accuracy. The results are listed in Table 2.1. From the table it can be seen that there is a
trade-off between accuracy and computation time. Another finding of the analysis of the cantilever beam is that also for larger deflections more GBEs are required in order to represent the true deflection.

The implementation in SimMechanics will be discussed in more detail in Chapter 4.

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Table 2.1: Simulated deflection of the HMA cantilever beam for different numbers of GBEs. The theoretical deflection is $d = 2.30\text{mm}$.

2.2 Other Methods

2.2.1 Euler-Bernoulli Beam Theory

A beam is defined as a structure that has one of its dimension much larger than the remaining two. Beam theories are often used at a pre-design stage to get a first insight into the behavior of structures and are also useful to validate computational solutions. There exists different beam theories based on various assumptions, i.e. different accuracies. The simplest and most useful was first described by Euler and Bernoulli. Following kinematic assumptions are made in the Euler-Bernoulli Beam Theory [11]:

1. The cross-section is infinitely rigid in its own plane.
2. The cross-section of a beam remains plane after deformation.
3. The cross-section remains normal to the deformed axis of the beam.

In case of transverse loads, i.e. pure bending, the transverse deflection $u$ of the beam is given by following differential equation:

$$\frac{d^2}{dx^2} \left( EI \frac{d^2u}{dx^2} \right) = p(x) \quad (2.4)$$

where $x$ is the coordinate along the beam and $p(x)$ is the distribution of transverse loading. This equation can be solved for different cases (e.g. clamped end), given four boundary conditions.
2.2.2 FEM

Beam theories lead to a solution with small calculation work for relative simple load conditions. If the load and the boundary conditions become complicated, numerical methods should be taken into account such as finite element analysis (FEA) software.

The finite element method (FEM) approximates solutions to boundary value problems. The solution domain is subdivided into elements of simple geometric shape, such as triangles, squares or tetrahedra [12]. Furthermore, a set of basis functions are defined such that each basis function is non-zero over a small number of elements only. This process is called discretization. The finite element space is defined as the set of all functions that can be written as linear combinations of the basis functions. The accuracy depends on the finite element space and the method used for computing the data.

This section exemplifies the difference between FEM and the lumped-parameter method with the uniform beam. Using the lumped-parameter method, the angle of deflection along the beam is discontinuous, i.e. changes from the \((i-1)\)-th GBE to the \(i\)-th GBE by \(\Delta \alpha_i\) (see Section 2.1.2). In the case of FEM, not only the deflection itself is set equal at the nodes but also the slope of the deflection curve is set continuous. Hence, each element has four boundary conditions (two on each node). As an approximation of the deflection curve an interpolation function (basis function) is introduced. E.g., a polynomial third degree would meet the requirements and fulfill the boundary conditions [4]. Finally, the system can be solved for the unknown coefficients of the interpolation function with all boundary conditions and Equation 2.4.
Chapter 3

Kinematic Model of Gripper

In this chapter, the kinematic model the HMA gripper will be presented. The gripper model consists of 63 GBEs and was implemented in SimMechanics.

3.1 Dimensions of the Gripper

The dimensions of the gripper are depicted in Figure 3.1. They are based on a real gripper as shown in Section 1.3.1 Figure 1.2. The gripper is divided into five parts: beam at the top, left and right beam and two tips. The width along the beams is assumed constant as well as the height (in direction perpendicular to the drawing). The tips were designed in CAD and a sketch is depicted in Appendix B.

![Dimensions of the gripper](image)

Figure 3.1: Dimensions of the gripper. The height (in direction perpendicular to the drawing) is assumed constant (7mm).

3.2 Mechanical Model

A mechanical model was designed based on the lumped-parameter method, shown on the right in Figure 3.2. A torsional spring-damper system is acting on each joint. The mass \( m \) of one GBE can be calculated by \( m = \rho V \), where \( V \) is the volume of one GBE and \( \rho = 0.98 \text{g/cm}^3 \) is the density of HMA at room temperature [13].
The moment of inertia $J$ is given by the formula for a cuboid-shaped object. The torsional spring stiffness $k$ is given by Equation 2.2. The top beam is represented by 11 identical cuboid-shaped GBEs of length $l_1$, mass $m_1$, moment of inertia $J_1$, torsional spring stiffness $k_1$ and damping coefficient $b$. The GBE in the middle (on the symmetrical line) is connected to the environment by a prismatic joint so that the gripper can be moved upwards and downwards. The right beam (equal to the left beam) is represented by 25 identical cuboid-shaped GBEs of length $l_2$, mass $m_2$, moment of inertia $J_2$, torsional spring stiffness $k_2$ and damping coefficient $b$. The reason for the different spring stiffness $k_2$ is the different chosen GBE length $l_2$. Finally, the tip is assumed as one rigid body of mass $m_t$ and moment of inertia $J_t$. This assumption is justified by claiming that the total compliance is mainly due to the top and left/right beam since the length of the tip is very small. The mass $m_t$ and the moment of inertia $J_t$ were calculated with CAD-Software with the density of HMA $\rho = 0.98 \text{g/cm}^3$.

The equivalent visualization of SimMechanics is shown on the left in Figure 3.2. The model consists in total of 63 GBEs: 11 GBEs on top, 25 GBEs on each side for the left and right beam and 2 tip GBEs.

![Figure 3.2: Left: visualization in SimMechanics. Right: equivalent mechanical model.](image-url)
Chapter 4

SimMechanics Implementation

This chapter presents the model of the gripper in SimMechanics including the interaction of the gripper with an object and proposes a method for the determination of the contact point.

4.1 Model Overview

The SimMechanics model of the gripper is shown in Figure 4.1. As discussed in Section 3.2 and illustrated on the left side, the GBE in the middle of the top beam (‘Fixed GBE’) is connected to the environment by a prismatic joint so that the gripper can be moved upwards and downwards. The top beam consists of the ‘Fixed GBE’ which is connected to ‘Right Upper Beam’ and ‘Left Upper Beam’. The ‘Right Vertical Beam’ and the ‘Left Vertical Beam’ are connected to a block called ‘Left/Right Object Interaction’ which models the reaction forces due to the interaction of the gripper with an object (discussed in Section 4.3). In this model, the object is a box (labeled yellow on the bottom left side). The block ‘Ground1’ represents the ground and is used for visualization reasons. For the up and down movement of the gripper a joint actuator is used which defines the velocity of the prismatic joint (shown on the left side).

4.2 Implementation of the Kinematic Model

As mentioned in Section 3.2, the beam is represented by a chain of alternating bodies and joints. The SimMechanics model of the right half of the upper beam is shown in Figure 4.2. One subsystem consists of a joint and a body block. The body block is shown in Figure 4.3. The body block has assigned three coordinate systems (CG, CS1, CS2). The center of gravity is defined by the CG coordinate system. CS1 and CS2 are used to connect the adjacent body blocks to the current body, and are defined on each end of the current body. All translation of the different coordinate systems are defined with respect to the adjoining CS (in that example CS1).

The joint subsystem consists of a revolute joint with a torsional spring-damper system. An initial condition block is also connected to the revolute joint in order to set the initial deflection to zero. Moreover, each body block has certain mass and inertia properties, i.e. \( m_1 \) and \( J_1 \) in the case of the upper beam which are given in Section 3.2.
4.3 Object Interaction

The object interaction turned out to be a difficult task since SimMechanics is not able to model contacts between bodies. The solver does not know the shape of a body in SimMechanics. Only the locations of the various coordinate systems are known. Therefore, the gripper contour is sampled into points which are tracked and checked whether the object is penetrated. Then a body actuator is used to apply an appropriate force. The object interaction is modeled by a soft contact model which means that the normal force $F_N$ is represented by a linear spring-damper system. Only if a contact point of the gripper penetrates the object, a force will be applied to this point. The contact force is divided into a normal and frictional force.

4.3.1 Normal Contact Forces

As mentioned above, a soft contact model is chosen to represent the object interaction. The normal force $F_N$ is governed by following equation:

$$F_N = \begin{cases} \ -k_n n - b_n \dot{n} & \text{if } -k_n n - b_n \dot{n} \geq 0 \text{ and } n \leq 0 \\ \ 0 & \text{else} \end{cases}$$

(4.1)

where $k_n$ is the spring constant and $b_n$ the damping rate. The penetration depth $n$ is locally perpendicular to the surface at the contact point. The normal velocity $\dot{n}$ is also perpendicular to the surface. Note that only a normal force $F_N$ will be applied if the gripper penetrates the object, i.e. if $n \leq 0$. In addition, the normal force $F_N$ is always positive to avoid sticking forces which implies the condition $-k_n n - b_n \dot{n} \geq 0$. This model introduces two additional parameters $k_n$ and $b_n$ which are typically chosen large.

The calculation of the normal force was implemented by conventional MATLAB Simulink blocks. With a body sensor the penetration depth $n$ and the normal velocity $\dot{n}$ are determined. The penetration depth is then multiplied with the spring constant $k_n$ and the normal velocity with the damping rate $b_n$. Afterwards, these two values are summed up and a saturation block guarantees that finally the normal force $F_N$ is always greater or equal than zero.
4.3. Object Interaction

Figure 4.2: Right upper beam composed of five subsystems. One subsystem consists of a torsional spring-damper system acting on the revolute joint and a body block. The ‘Joint Initial Condition’ block sets the initial relative angle between the elements to zero.

4.3.2 Frictional Contact Forces

Frictional forces can be divided into static and kinetic friction. To do so, the tangential component of the contact force has to be known since static friction occurs if two objects are not moving relative to each other and the frictional force \( F_F \) is governed by the equation:

\[
F_F \leq \mu_s F_N
\]  

(4.2)

where \( \mu_s \) is the coefficient of static friction. Given that the relative velocity \( v_{rel} \) between the objects is zero, the objects start moving relative to each other as soon as the absolute value of \( F_F \) is greater than \( \mu_s F_N \). In that case, the frictional force \( F_F \) is given by:

\[
F_F = \mu_k F_N
\]  

(4.3)
where $\mu_k$ is the coefficient of kinetic friction.
Unfortunately, SimMechanics body sensors are not able to measure forces and therefore it is not possible to distinguish between static friction and kinetic friction. For that reason, a linear regularized friction model was used as shown in Figure 4.4. The idea is that the inequality of static friction (Equation 4.2) is replaced by a linear function with a very steep slope described by the parameter $c$. An advantage of this approach is that now the frictional force $F_F$ is fully described by the relative velocity $v_{rel}$ only. The relative velocity $v_{rel}$ is measured by a body sensor. The function in Figure 4.4 assumes that the coefficient of static friction is equal to the coefficient of kinetic friction, i.e. $\mu = \mu_s = \mu_k$, which could easily be modified if the coefficients do not coincide.

The coefficient of static friction $\mu_s$ was experimentally determined between HMA and different materials and is shown in Figure 4.5. Depending on the object’s material, this value can be modified in the SimMechanics model.

Figure 4.5: Experimentally determined coefficients of static friction $\mu_s$ between HMA and different materials.

### 4.3.3 Determination of the Contact Point

As mentioned above, the solver does not know the shape of a body in SimMechanics. Thus, the contour of the gripper’s tip is sampled into 11 contact points on each side as depicted in Figure 4.6. All coordinate systems are perpendicular to the contour so that the normal force $F_N$ is applied in $x$-direction and the frictional force $F_F$ in $y$-direction. The concept of the determination of the contact point is shown in Figure
4.7. For the sake of clarity only 2 sample points are drawn. The world reference frame is labeled with the axis $x_W$ and $y_W$. The sample points are tracked with respect to the world frame as vectors $\vec{p}_1(t)$ and $\vec{p}_2(t)$, respectively. The object itself has a continuous contour and is described by a function $f(x_W)$, also with respect to the world frame. Knowing the positions of the sample points, a test function checks whether or not a point penetrates the object with the help of the function $f(x_W)$. Then, normal force and frictional force are applied as soon as a point is inside the object by computing the penetration depth $n$ and normal velocity $\dot{n}$.

Figure 4.6: Sampled contour of the gripper. On each side 11 contact points are defined. The coordinate systems are perpendicular to the contour.

Figure 4.7: Interaction of the gripper with an object. The object is defined as a continuous function $f(x_W)$ with respect to the world frame. The gripper contour is sampled into several points. For the sake of clarity only two points are shown. These two points are tracked with respect to the world frame $(\vec{p}_1(t), \vec{p}_2(t))$. As soon as a point on the contour penetrates the object, normal force and frictional force are applied.
Chapter 5

Simulation and Results

The interaction of the gripper with two different objects (a cylinder and a box) was simulated and then compared with a real-world experiment.

5.1 Interaction with Objects

5.1.1 A Cylinder Shaped Object

A cylinder with radius 15mm was the first object for testing the interaction with the gripper. An advantage of a cylinder-shaped object is that the contour can be described by only one function (Equation of the Circle). The interaction was simulated with a downward and upward speed of 4mm/s. The solver configuration is listed in Table 5.1. A stiff ODE-solver turned out to perform best and therefore the ode23s (stiff/Mod. Rosenbrock) was chosen. The parameter m-file can be found in the Appendix A.1.

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</tr>
<tr>
<td>Absolute tolerance</td>
<td>1e-3</td>
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</tbody>
</table>

Table 5.1: Solver configuration.

It should be noted that the influence of the friction is significant. As can be seen from Figure 5.1, both snapshots were taken at the same time step. On the left side the contact is frictionless whereas on the right side the contact is modeled with a coefficient of friction \( \mu = 0.7 \). With friction, the gripper starts buckling which was also observed for a real gripper.

5.1.2 A Box Shaped Object

As a second object a square box of dimensions 28mm × 28mm was chosen. The box is a bit more complicated compared to the cylinder since it requires more than one function to describe its contour. Hence, the determination of the contact point needs to check whether the point is in contact with the top surface, side surface or the rounded corner.
Figure 5.1: Comparison between a frictionless contact and a contact with $\mu = 0.7$. The snapshots were taken at the same time step.

Figure 5.2 shows a snapshot series of the interaction with the box. The coefficient of friction in this series is $\mu = 0.6$. After touchdown ($\approx 1$ s) the gripper envelopes the box and starts buckling. As soon as the tip is in contact with the side surface ($\approx 3$ s) this buckling behavior stops and the right and left beam bend outwards.

Figure 5.2: Snapshot series of the interaction with the box. The coefficient of friction is $\mu = 0.6$.

5.2 Validation of Linear Stress-Strain Relation

The lumped-parameter method proposed in Section 2.1 assumes a linear relation between stress and strain, described by the Young’s modulus $E$. Hence, this approach is only applicable as long as stress and strain are in this linear region. This section shows that the following simulation results are valid because the estimated stress-strain from bending measurements falls into the linear region of HMA. Note that once leaving this linear region, the model could be extended if the stress-strain relation is known. This relation can be experimentally determined as presented by Brodbeck et al. [1] where strain and stress of an HMA string were measured when variations of forces were exerted, shown in Figure 5.3. The stress-strain relation is linear if the strain is smaller than 0.2 or accordingly the stress is smaller than $1.2 \cdot 10^6$ Pa.

In SimMechanics, the bending moments were measured with a joint sensor for both the cylinder and the box. The largest measured bending moments are listed in Table 5.2.

Using the relation between bending moment $M$ and stress $\sigma$:

$$\sigma = \frac{M}{I} \cdot z \quad (5.1)$$

where $z$ is the coordinate along the cross section, the maximum stress can be calculated and will be at the top surface ($z_{\text{max}}$). These values are also listed in Table 5.2.

It is obtained that the resulting stress is significantly smaller than the maximum stress of $1.2 \cdot 10^6$ Pa for the linear region. Thus, the linear stress-strain relationship is valid.
5.3 Results

5.3.1 Linear Actuator Force

An interesting aspect of designing a passive compliant gripper may be the required linear actuator force in order to pick up an object. The gripper geometry could then be optimized in a second step to reduce the maximum force that has to be applied. Figure 5.4 shows measurements of the reaction force of the prismatic joint for the cylinder and the box when the gripper moves towards the object with velocity 4 mm/s and a traveling distance of 28 mm. Initially, the actuator force \( F_{\text{act}} \) is negative since the weight of the gripper has to be supported. Afterwards, it increases in both cases and reaches a maximum at a certain point. At time 7s the gripper has reached the lowest position.

Figure 5.3: Tension test of an HMA string from Brodbeck et al. [1]. Linear relation until approximately \( \sigma_n = 1.2 \cdot 10^6 \) Pa.

### Table 5.2: Largest bending moment and resulting stress for box and cylinder.

<table>
<thead>
<tr>
<th></th>
<th>Largest bending moment [Nm]</th>
<th>Resulting stress [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box</td>
<td>( 5.41 \cdot 10^{-3} )</td>
<td>( 2.9 \cdot 10^5 )</td>
</tr>
<tr>
<td>Cylinder</td>
<td>( 3.82 \cdot 10^{-3} )</td>
<td>( 2.0 \cdot 10^5 )</td>
</tr>
</tbody>
</table>

Figure 5.4: Linear actuator force for cylinder/box (\( \mu = 0.6 \)).
5.3.2 Angle of Deflection

To validate the model, the gripper was mounted on a robot arm and moved towards the exact same sized objects as discussed above. The angle of deflection $\phi(t)$ is defined as the angle between the top beam and the right and left beam, respectively. In the simulation these two angles coincide since the geometry is symmetric. The initial angle of deflection $\phi_0 = \phi(t = 0)$ is shown on left in Figure 5.6. The simulated angle over time is shown in the graphs of Figure 5.5. Again, the gripper reaches the lowest point at 7s and moves with velocity 4mm/s. This values for $\phi(t = 7\text{s})$ are listed in Table 5.3 in the column ‘Simulated angle’. The same angle was visually measured from the experiment, shown on the right in Figure 5.6. The angle is determined on both left and right side because the experiment is not perfectly symmetric. These values and its averages are also listed in Table 5.3 and compared with the simulated values which gives a relative error for the box of 1.9% and for the cylinder of 1.5%.

![Figure 5.5: Angle of deflection $\phi(t)$ for cylinder/box ($\mu = 0.6$).](image)

<table>
<thead>
<tr>
<th>Real gripper experiment</th>
<th>Left angle</th>
<th>Right angle</th>
<th>Average</th>
<th>Simulated angle</th>
<th>Relative error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box</td>
<td>112.1°</td>
<td>106.9°</td>
<td>109.5°</td>
<td>107.4°</td>
<td>1.9</td>
</tr>
<tr>
<td>Cylinder</td>
<td>110.3°</td>
<td>106.8°</td>
<td>108.6°</td>
<td>107.0°</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 5.3: Angle of deflection during picking up for real world experiment and simulation.
Figure 5.6: Left: definition of the angle of deflection $\varphi_0$ at time 0s. Right: angle of deflection $\varphi(t = 7s)$ visually measured for the box and the cylinder at the lowest point.
Chapter 6

Future Work

6.1 Force Closure Analysis

The current model does not consider whether or not the force closure of the gripper is sufficient to pick up the object. This constraint could be implemented by knowing the normal force $F_N$ and the frictional force $F_F$ before lifting the object. These forces can be measured in SimMechanics.

The contact forces acting on the cylinder and the box are depicted in Figure 6.1. The lowest point of the gripper has no influence on the force composition in the case of the box whereas for the cylinder the lowest point determines the direction of the contact forces. The contact point of the cylinder is described by an angle $\alpha$. Adding up the force vectors results in a lifting force $F_{\text{lift, cyl}}$ in $y$-direction of:

$$F_{\text{lift, cyl}} = 2F_N \sin \alpha + 2\mu F_N \cos \alpha \quad (6.1)$$

For the box, only the frictional force has a component in $y$-direction and therefore the lifting force $F_{\text{lift, box}}$ is given by:

$$F_{\text{lift, box}} = 2\mu F_N \quad (6.2)$$

The gripper is able to pick up the object if the lifting force is greater than the weight force of the object, i.e. $F_{\text{lift, box}} > m_{\text{box}} \cdot g$ and $F_{\text{lift, cyl}} > m_{\text{cyl}} \cdot g$, where $m_{\text{box}}$ is the mass of the box, $m_{\text{cyl}}$ is the mass of the cylinder and $g$ is the gravitational acceleration, respectively.

![Figure 6.1: Contact forces acting on the cylinder and box.](image)

6.2 Design Optimization

Having a gripper model, this model can be used to run an optimization algorithm to get a new designed gripper which may have better properties regarding the process.
of picking and placing a certain object. To do so, the model has to be generalized to arbitrary parameters, e.g. the length of the beams, the angle between the beams etc.

6.3 Auto-Detection of Object Contour

Further developments of the robot itself could also be a continuation of this project. This includes, for example, an auto-detection device for perceiving the object contour. By mounting a camera on the robot, following task could be accomplished: the robot has to pick and place a certain object. First, the camera detects the object contour and above mentioned optimization algorithm produces an optimal gripper for that specific object. Afterwards, the fully automated glue supply mechanism manufactures this optimal gripper and finally the robot is able to fulfill its task.

6.4 3D Modeling

The model may be extended to 3D. In that case, the lumped-parameter method may not be suitable anymore and other methods like FEM should be considered. For the grippers that are currently used at the BIRL, 2D modeling is sufficient since the movement of the gripper is expected in only one plane. For other types of grippers this may not be the case anymore.
Appendix A

MATLAB Code

A.1 Parameters.m

The Parameters.m file has to be executed before running the model file.

```matlab
rho = 0.98; % [g/cm^3]
height = 0.7; % [cm]
E = 8.9e6; % [N/m^2] Young's modulus HMA
b = 0.01; % damping coefficient
angle_beam = 7; % [degree]

% Top
l1 = 0.21; % [cm]
w1 = 0.4; % [cm]
m1 = l1*w1*height*rho; % [g]
Iz1 = 1/12 *w1*height*(0.01*w1)^3; %[m^4]
k1 = E*Iz1/(0.01*l1); %spring [N*m/rad]
J1 = diag([0, 0, 1/12 *m1*(l1^2 + w1^2)]);

% Drawing
l1 = 2.1; % [mm]
w1 = 4; % [mm]
```

```matlab
CS1_r = [0, 0, 0];
CG_r = [l1/2, 0, 0];
CS2_r = [l1, 0, 0];

CS2_r_adj = [l1 - w1/2, -w1/2, 0];
CS2_l_adj = [-l1 + w1/2, -w1/2, 0];

CS1_l = [0, 0, 0];
CG_l = [-l1/2, 0, 0];
CS2_l = [-l1, 0, 0];
```

% Left Beam = Right Beam
m_2 = 1.2*w_2*height*rho; % [g]
I_z2 = 1/12 *0.01*height*(0.01*w_2)^3; %[m^4]
k_2 = E*I_z2/(0.01*l_2); %spring [N*m/rad]
J_2 = diag([0, 0, 1/12 *m_2*(l_2^2 + w_2^2)]);

%% Tip
A_t = 112*0.01; % [cm^2]
m_t = A_t*height*rho; % [g]
I_t = 1/12*0.01*height*(0.01*w_2)^3; %[m^4]
k_t = E*I_t/(0.01*l_2); %spring [N*m/rad]
J_t = diag([0, 0, J_zz]);

%% CG
CG_t = [8.808687272, -2.331736882, 0]; % from CAD

%% Boundaries
% Define coordinate systems on tip
CS_t1 = [5.150757595, -10.020815280, 0];
CS_t1_o = [0, 0, -135];
CS_t2 = [12.289596972, -5.710403028, 0];
CS_t2_o = [0, 0, -45];
CS_t3 = [7.791847200, -10.020815280, 0];
CS_t3_o = [0, 0, -45];
CS_t4 = [6.564971157, -10.606601718, 0];
CS_t4_o = [0, 0, -90];
CS_t5 = [14.443333542, -3.55666458, 0];
CS_t5_o = [0, 0, -45];
CS_t6 = [13.364995700, -7.864199570, 0];
CS_t6_o = [0, 0, -45];
CS_t7 = [11.214198244, -6.785801756, 0];
CS_t7_o = [0, 0, -45];
CS_t8 = [10.135860402, -7.864199570, 0];
CS_t8_o = [0, 0, -45];
CS_t9 = [9.057522561, -8.942477439, 0];
CS_t9_o = [0, 0, -45];
CS_t10 = [7.330338022, -10.454360783, 0];
CS_t10_o = [0, 0, -45 - 22.5];
CS_t11 = [5.799604293, -10.454360783, 0];
CS_t11_o = [0, 0, -135 + 22.5];
CS_t12 = [16.60009224, -1.399990776, 0];
CS_t12_o = [0, 0, -45];
CS_t13 = [17.031748681, 0.770510047, 0];
CS_t13_o = [0, 0, -45 + 67.5];
CS_t14 = [17.138007330, -0.414256067, 0];
CS_t14_o = [0, 0, -45 + 32.75];

% Dimension in mm for drawing
l_2 = 1.72; % [mm]
w_2 = 4; % [mm]
CS1.b = [0, 0, 0];
CS2.b = [1.2, 0, 0];

CG.b = [l_2/2, 0, 0];

height = 7; % [mm]

k_Ground = 10000; % [N/m]

b_Ground = 1; % [Ns/m]

mu_Ground = 0.6;

c = 10e-10; % [m/s]
Appendix B

CAD Drawing

B.1 Tip of the Gripper

The CAD drawing of the gripper’s tip is shown in Figure B.1:

Figure B.1: CAD drawing of the tip. All dimension in mm.
Bibliography


