Formation Control: Uncertain Distances and Nonrobust Behavior

Brian D.O. Anderson
National ICT Australia
Australian National University

Keith Glover Workshop on Uncertain Systems

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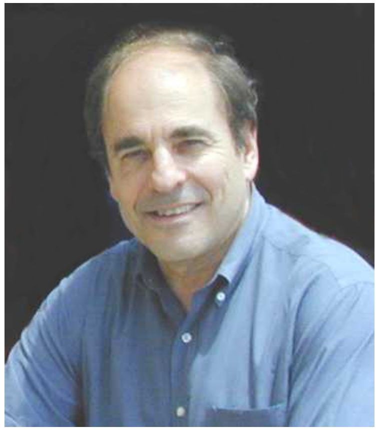
Joint work with

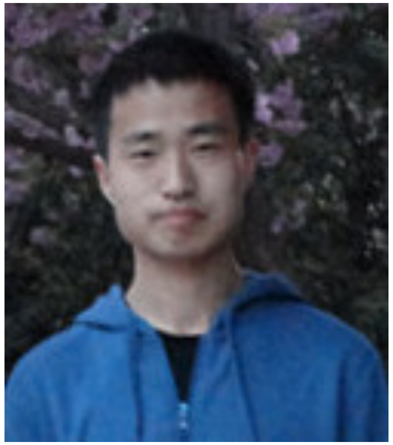


A. Stephen Morse

(Yale)

Shaoshuai Mou

(Yale)



Zhiyong Sun

(ANU)

DEDICATED TO:

Best example of nonrobust behavior: Keith learning how to sailboard on Lake Burley Griffin in Canberra, while simultaneously working out error bounds for transfer function Hankel norm approximation.

DEDICATED TO:

- He will become an **emeritus professor.**
- **Emeritus** is a Latin word: **E + meritus**
- “e”= out
- (in the sense of kicked out)
- “meritus” –because he deserved it.
- But, unlike some past Chancellors, he won’t be beheaded.

OUTLINE

- Background and problem description
- Motion equations
- Main results
- Properties of the movement
- Additional remarks and conclusion

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Background-I

- **Formation control:**
  - Common tasks are moving formations of UAVs from point A to point B, having them circle point C, etc.
  - Maintaining the formation shape may be important, e.g. for optimum target localization, or area coverage.
  - Shape maintenance and moving the whole formation are separable tasks. This talk considers a shape maintenance question.

L. Krick, M. E. Broucke, and B. A. Francis, "Stabilisation of infinitesimally rigid formations of multi-robot networks," *International Journal of Control,* vol. 82, pp. 423-439, 2009.

Background-II

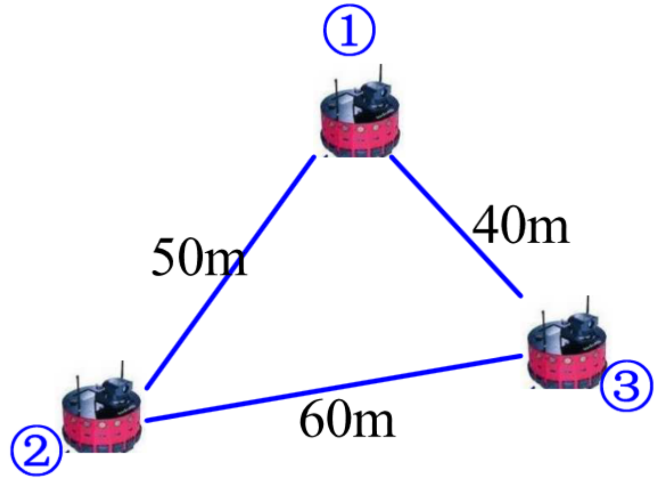
- **Formation shape control:**
  - By controlling a certain set of distances, the whole formation shape will be maintained (graph rigidity theory).
  - Each agent moves cooperatively to achieve the desired inter-agent distances (gradient based control).
  - The accuracy in measuring the actual distances is crucial for the final formation shape.

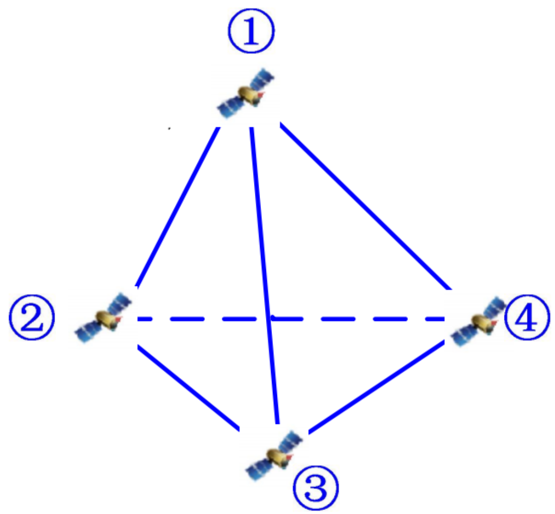
L. Krick, M. E. Broucke, and B. A. Francis, "Stabilisation of infinitesimally rigid formations of multi-robot networks," *International Journal of Control,* vol. 82, pp. 423-439, 2009.

Background and problem description

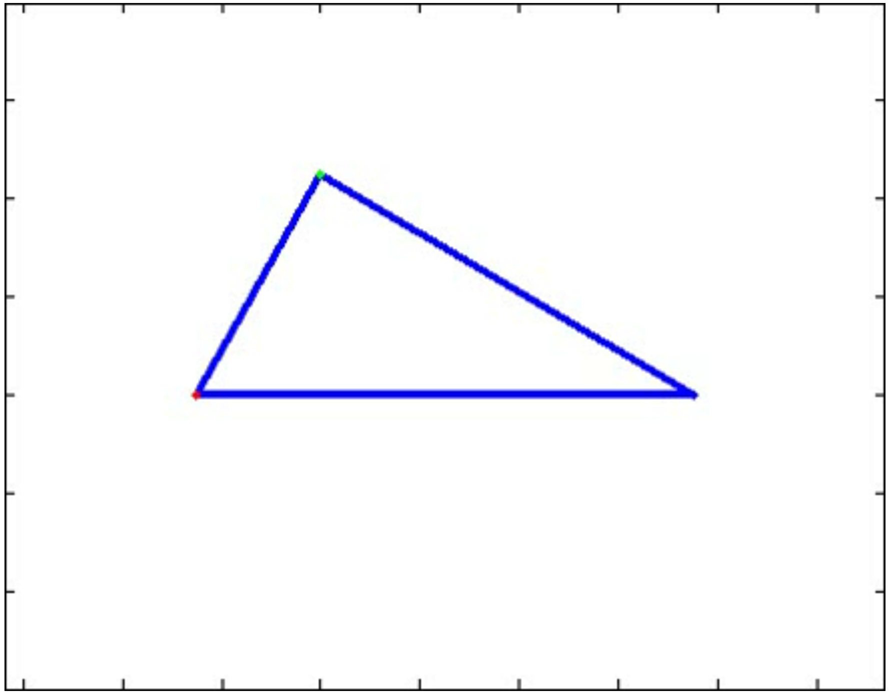
What will happen if

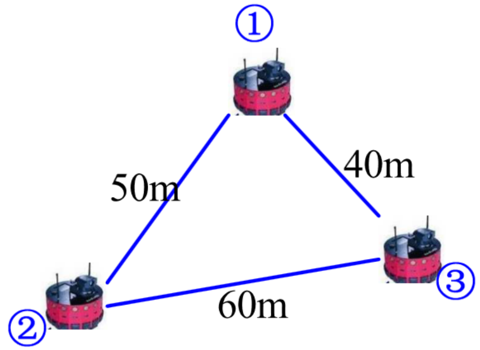
- There exist *unequal biases in the measurements* of the same distance between two joint agents?
- Or equivalently, two joint agents have *different views of the same distance*?
- E.g., agent 1 wants to be *50m* from agent 2 and thinks it is *49m* away; agent 2 wants to be *50m* from agent 1 and thinks it is *51m* away.
- Agents *may not be aware of the mismatches,* which may come from measurement errors, actuator deviations, etc.





Previous results on 2-D space case



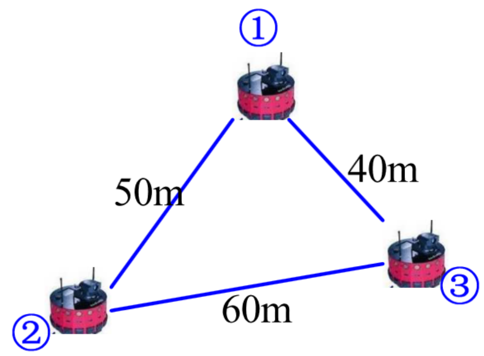


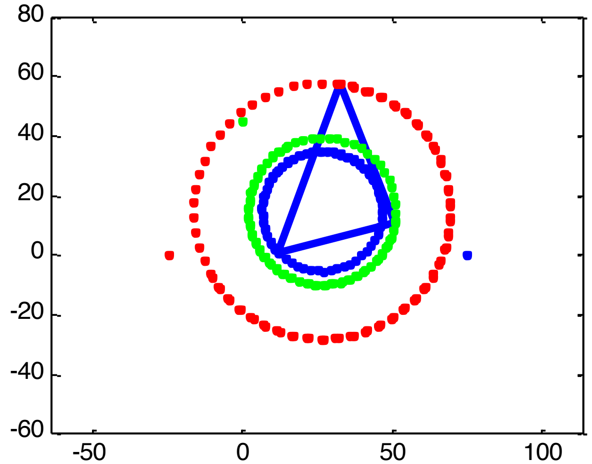
Simulation on 2-D triangle formation shape with distance mismatch

A. Belabbas, S. Mou, A. Morse, and B. Anderson, "Robustness issues with undirected formations," in Decision and Control (CDC), 2012 IEEE 51st Annual Conference on, 2012, pp. 1445-1450.

Previous results on 2-D space case

- Instead of staying stationary with the correct shape, the formation will generally exhibit a circular motion in the plane, and the shape will be approximately correct.
- All the agents will rotate about the same point.
- The angular velocity depends on the mismatch values as well as on the final shape.
- The radius may be large even with very small errors.





A. Belabbas, S. Mou, A. Morse, and B. Anderson, "Robustness issues with undirected formations," in Decision and Control (CDC), 2012 IEEE 51st Annual Conference on, 2012, pp. 1445-1450.

What this talk is mainly about

We describe what happens for a three-dimensional rather than two-dimensional formation when there are small distance mismatches.

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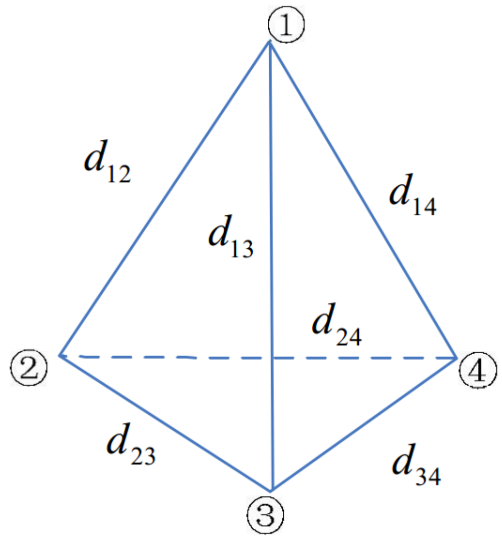
We start with a tetrahedron formation:

- Each distance is maintained by two joint agents (undirected formation);
- Some agent pairs may have differing views about the same distances;
- Gradient control law is employed.



The potential functions:

$$V(p_1, p_2, p_3, p_4) = \frac{1}{4} \sum_{1 \le j < i \le 4} [\|p_i - p_j\|^2 - d_{ij}^2]^2$$

$p_i \in R^3$ is the position of agent i,

d_{ij} is the desired distance between agents i and j,

$z_{ij} = p_i - p_j$ is the relative position between agents i and j,

$e_{ij} = \|z_{ij}\|^2 - d_{ij}^2$ is the distance square error

Motion equations: no unequal biases

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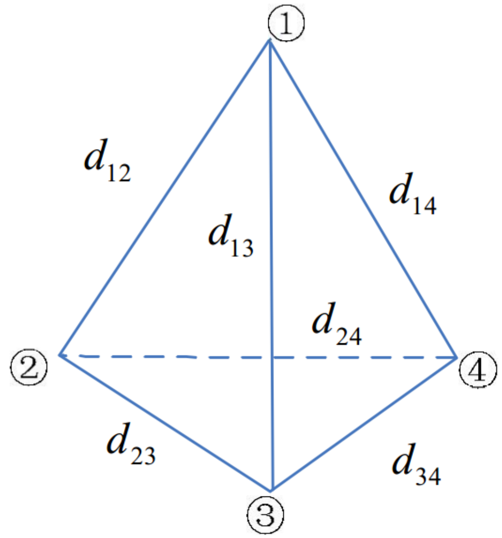
$$\dot{p} = -\nabla V(p)$$



Easy modification (not recorded here) when there is a mismatch!

There are two other sets of equations, which involve the *relative positions and distance square errors.*

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Main Results: stability of the error system with no mismatch

Recall the definition of the distance error: $e_k = \|z_k\|^2 - d_k^2$

The distance error system: $\dot{e} = -2\underline{R(z)}R^T(z)e$

Rigidity matrix

A key point:

The distance error system is self-contained:

$$\dot{e} = \underline{A(e)}e$$

Matrix A depends smoothly on e

- The nominal error system converges *exponentially fast*.

Main Results: stability of the error system with unequal distance biases

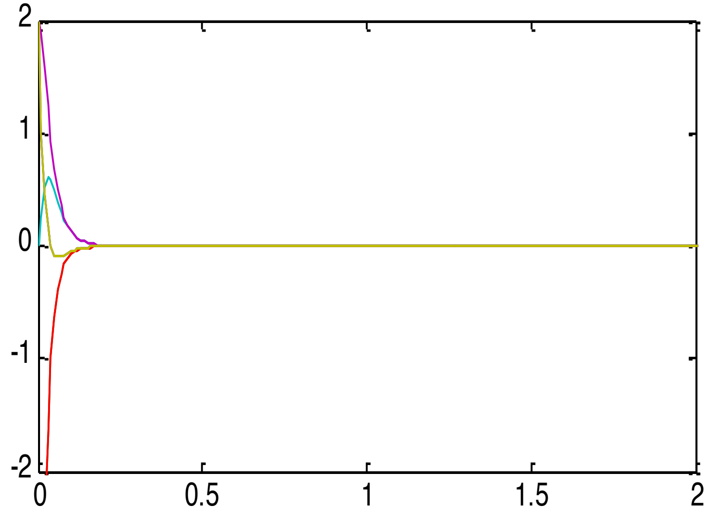
If there exist unequal biases in the perceived distances, the distance error system is modified as

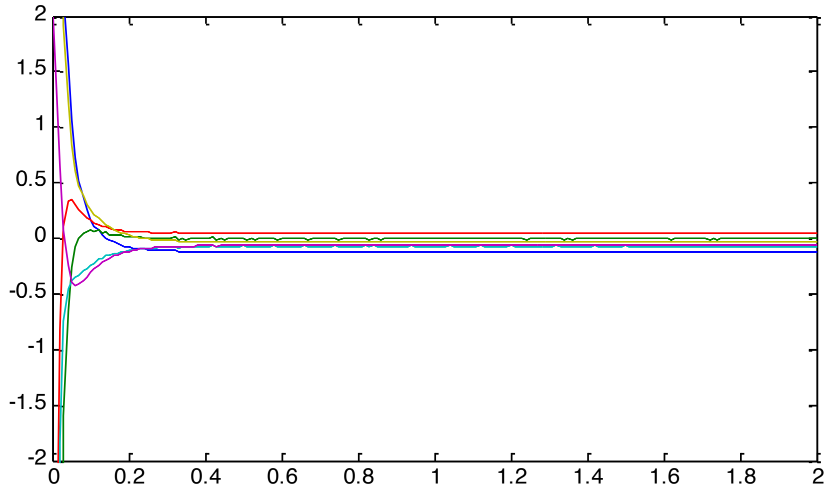
$$\dot{e} = A(e)e + B(e)\underbrace{\mu}_{\textbf{Distance mismatch}}$$

- The nominal error system (without mismatch term) converges *exponentially fast.*
- Due to the exponential stability, the error system with small distance mismatch term will *converge to an equilibrium close to the origin*.

The final (t goes to infinity) formation shape will be a slightly distorted one as compared to the desired one.

Main Results



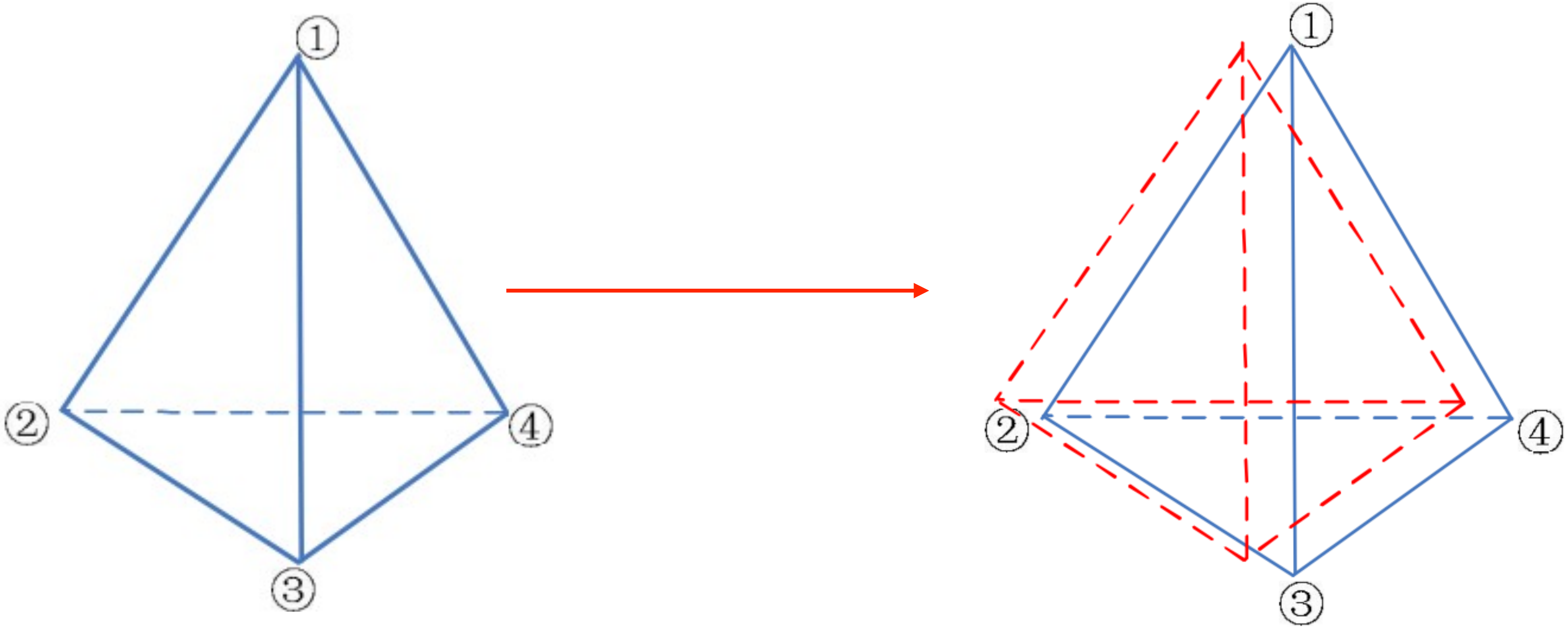


Convergence of the distance square errors

Left: the error system without distance mismatch

Right: the error system with distance mismatch

Main Results: the nice aspect



A slightly distorted formation shape

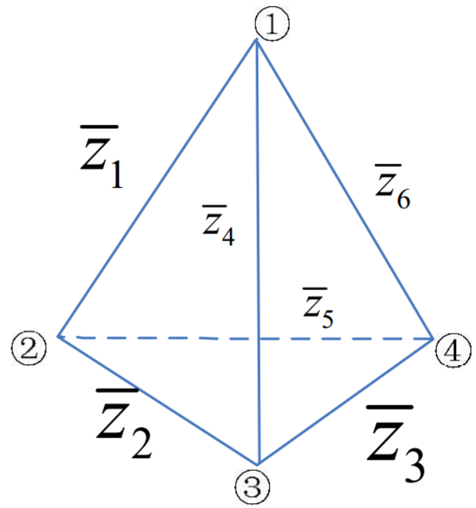
Main Results: the surprise

The final shape is fixed, *but additional motion will occur.*

The relative position is defined as $z_{k_{ij}} = p_i - p_j$

The reduced order $\bar{z}$ (relative position) system when shape is fixed:

$$\frac{d}{dt}(\bar{z}_1, \bar{z}_2, \bar{z}_3) = (\bar{z}_1, \bar{z}_2, \bar{z}_3)F$$

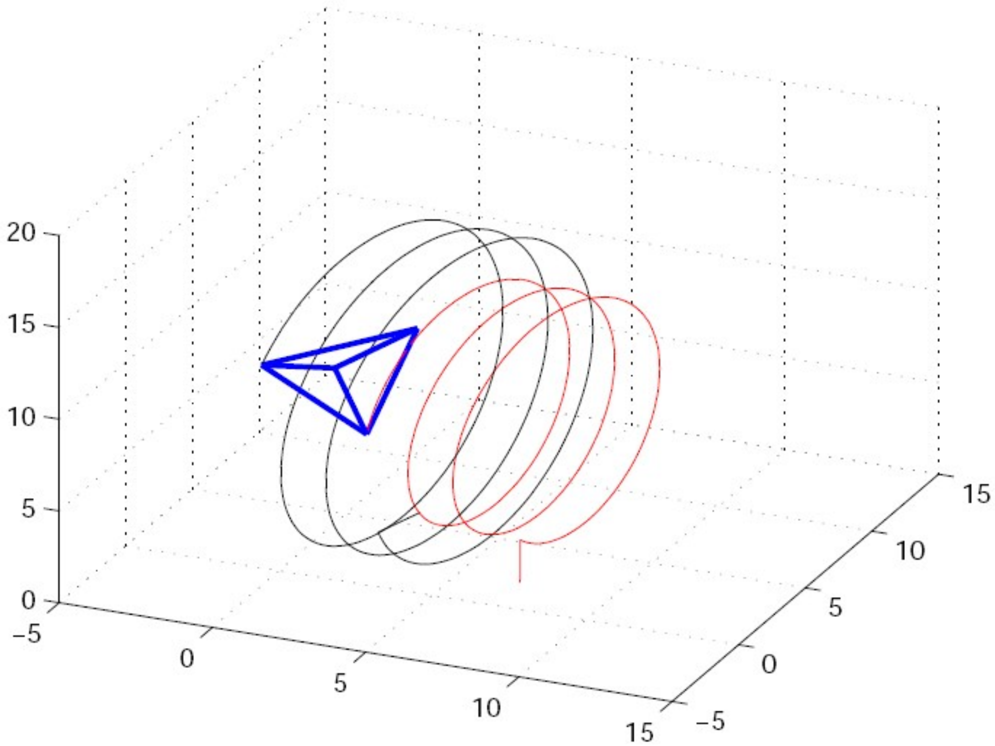


- The matrix F has two pure imaginary eigenvalues and one zero eigenvalue.

  The movement is a combination of translation and rotation!

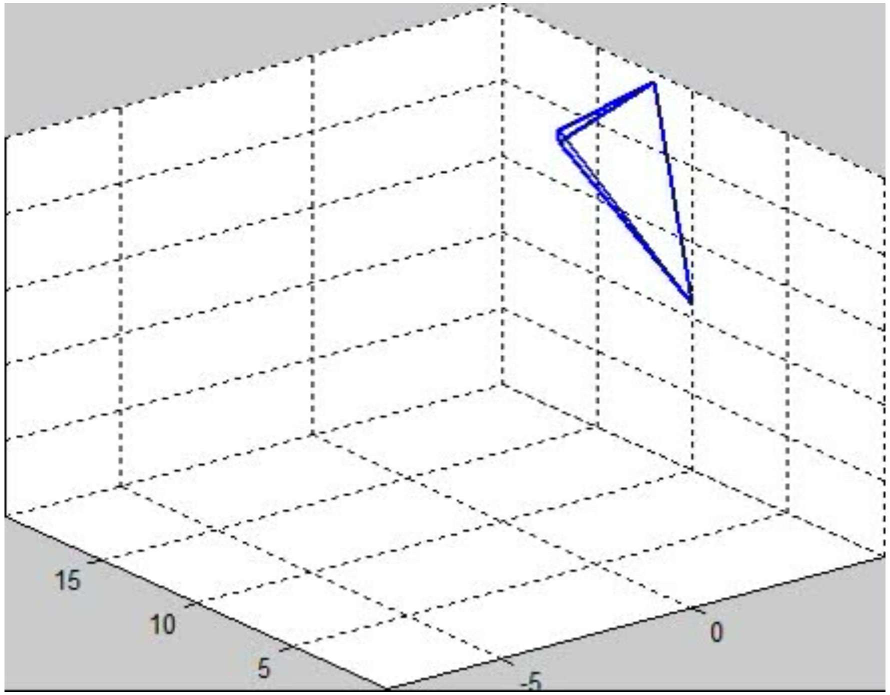
- Or in special circumstance: the matrix has three zero eigenvalues--- translation movement.

Main results: simulation on agents' movement



The agents will undergo additional movements…

Main results: simulation on agents' movement



The agents will undergo additional movements…

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Properties of the movement

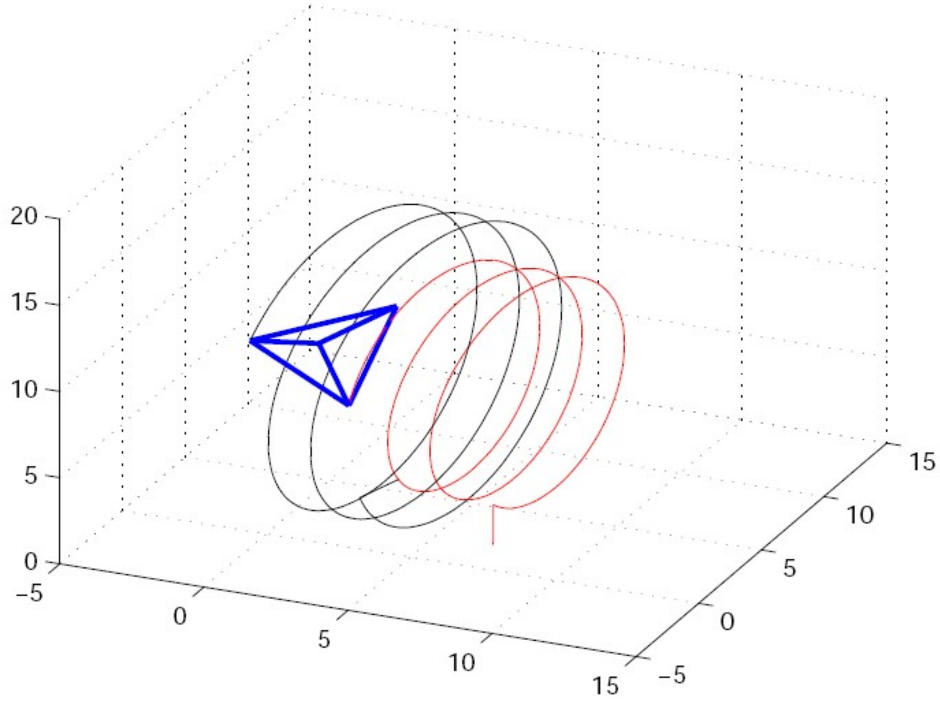
The movement is a helix.

Recall that the movement is generally *a combination of rotation and translation*. Agent i's motion can be described as:

$$\dot{p}_i = \underbrace{\alpha_i \cos \omega t + \beta_i \sin \omega t}_{\text{Rotation term}} + \underbrace{\delta_i}_{\text{Translation term}}$$

- ***The norm of each agent's speed is constant (nontrivial fact). This implies (again nontrivial):***
- ***The axis about which the agent rotates is the same as the direction of the translation—a helical movement.***
- ***When*** $\omega = 0$***, this indicates a translation-only movement.***

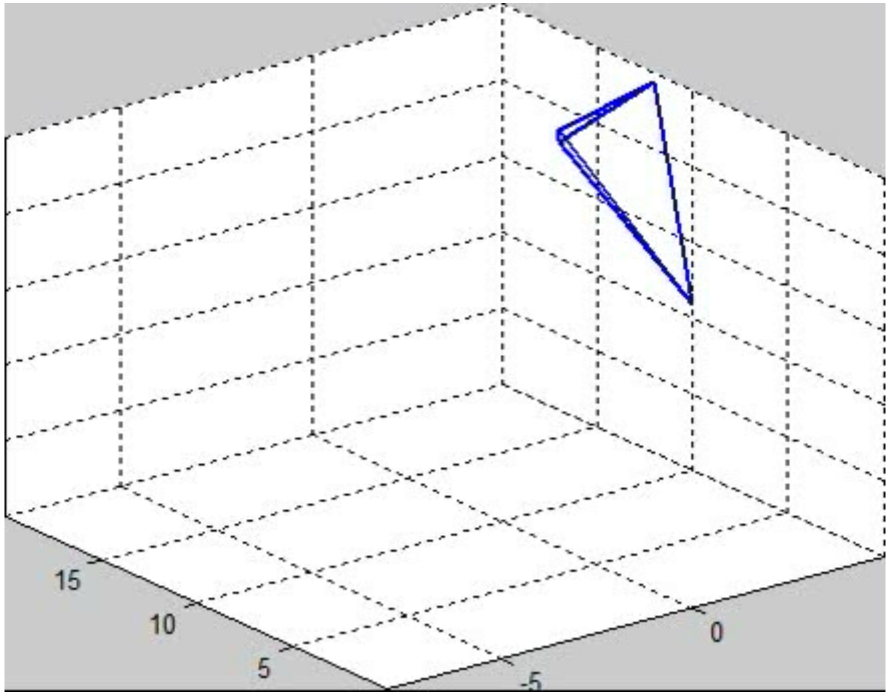
Look again: agents' movement

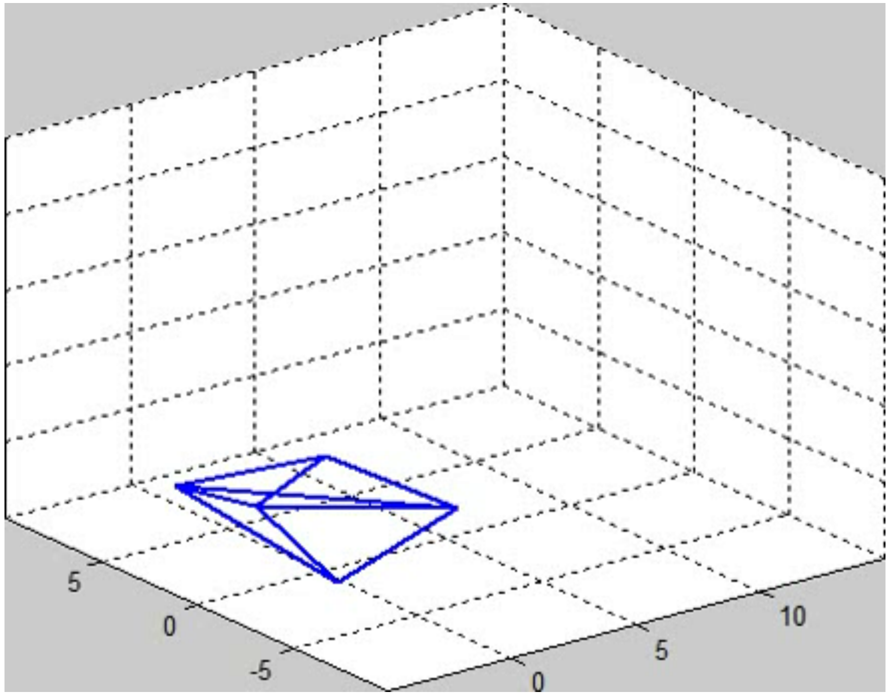


A typical helical movement

The axis about which the agent rotates is the same as the direction of the translation

Look again: agents' movement





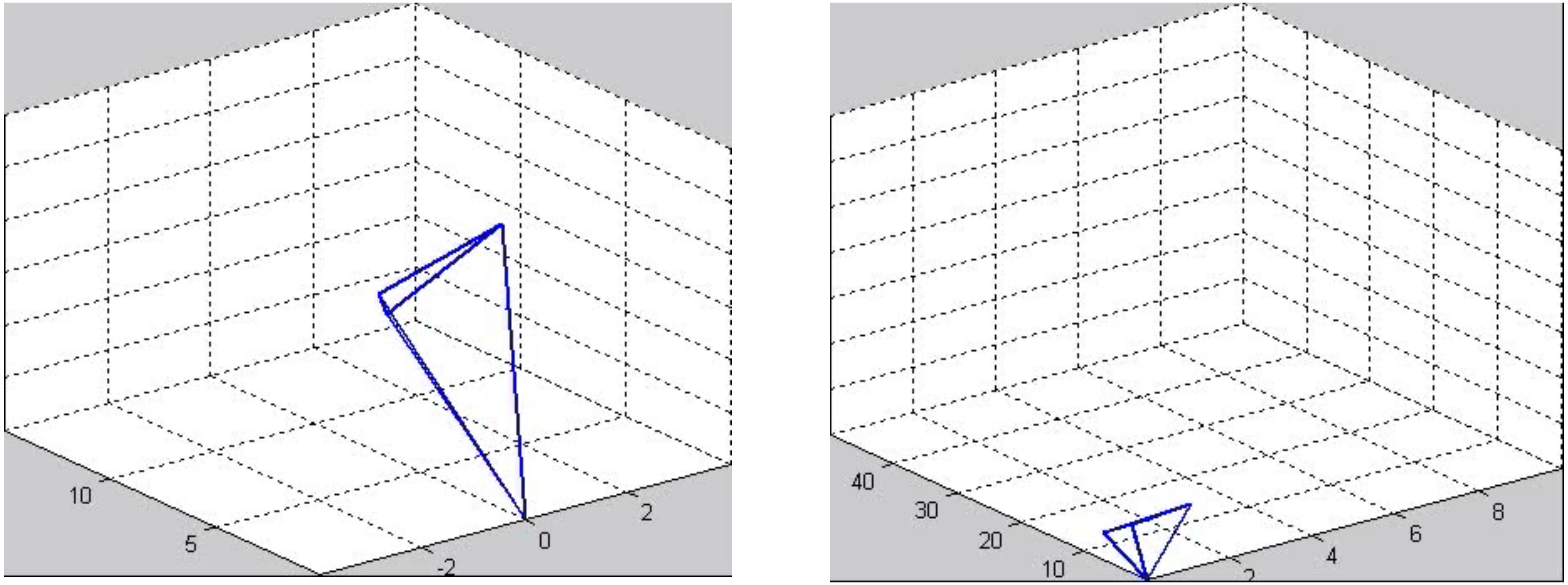
A typical helical movement

(Left: single tetrahedron shape; Right: double tetrahedron shape)

Properties of the movement

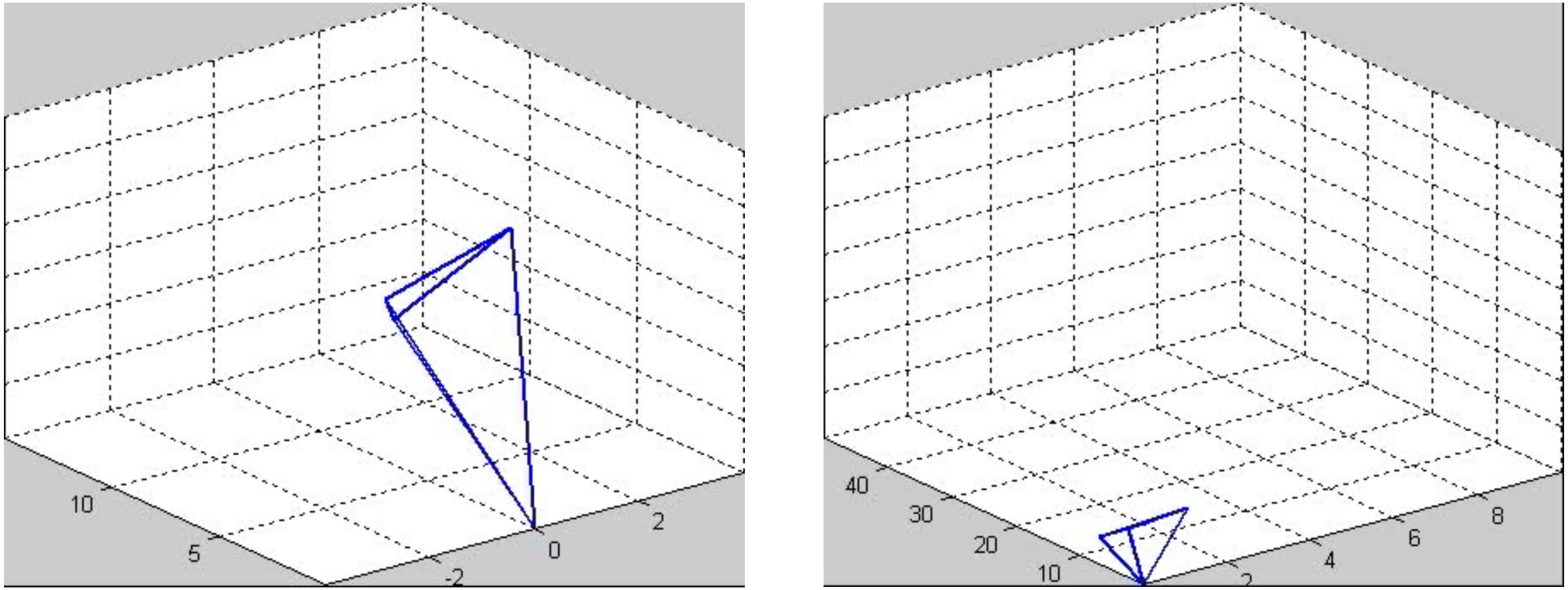
- The *translational direction* depends on the initial positions.
- The translational speed and the rotational velocity are related to the mismatches, but are independent of the initial positions.
- The *rotational velocity* depends on *the formation shape* as well as on *the distribution of mismatches in each face*.
- *Rotation-only or translation-only* movement can be observed as special cases.
- We do not have a complete set of nice formulas but now understand how to derive them.

Properties of the movement



Simulations on rotation-only and translation-only movements

Properties of the movement



Simulations on rotation-only and translation-only movements

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Additional remarks and conclusion

Some ideas on how to stop the undesired motions

- **Directed formation setup:**
  Each edge length is maintained by a single agent.

- **Add a dead zone (the tolerance domain) in the distance errors:**
  The formation stops moving when distance errors enter into the domain.

- **The estimating agent approach:**
  Choose an agent in each edge as the *estimating agent* and design an *adaptive estimation law* to estimate the mismatch.

Think positively: One may take advantage of the mismatch to steer the formations.

Conclusion

Uncertain systems: robust aspects

- The *exponential stability* of the error system guarantees the convergence of the edge lengths of the formation shapes.
- *The final formation shape will not diverge.*

Uncertain systems: nonrobust behaviors

- The agents' motion *will not come to rest* due to distance mismatch.

- In 2-D case, all the agents *will exhibit a circular orbit in the plane* (the translation-only movement is non-generic).

- In 3-D case, all the agents *will undergo a helical movement* (the translation-only movement or the rotation-only movement is non-generic).

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Thanks!