Loop shaping, localized behaviors, and multiresolution feedback systems

Rodolphe Sepulchre -- University of Cambridge

Gloverfest, September 2013

A special talk for a special occasion

- What am I allowed to say to celebrate Keith's achievements?
- What am I allowed to say in Jan's slot?

Two great achievements of LTI theory deeply missed in nonlinear systems theory

- Loop shaping: a concept to handle feedback
- Behaviors: a mathematical language to distinguish between latent and manifest variables

A generalization through state-space models proved to be difficult (intractable).

The behavior of a nonlinear electrical circuit

J. Physiol. (1952) 117, 500–544

A QUANTITATIVE DESCRIPTION OF MEMBRANE CURRENT AND ITS APPLICATION TO CONDUCTION AND EXCITATION IN NERVE

By A. L. HODGKIN and A. F. HUXLEY

From the Physiological Laboratory, University of Cambridge

(*Received* 10 *March* 1952)

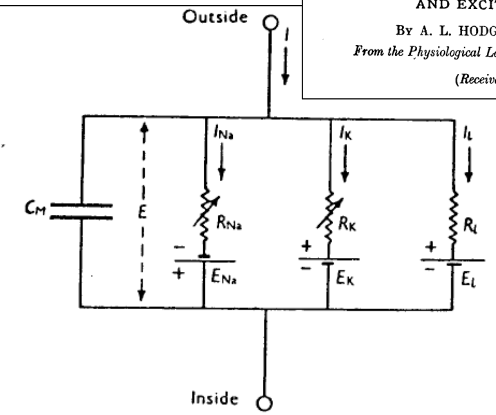
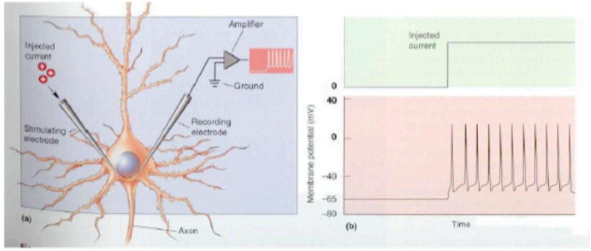
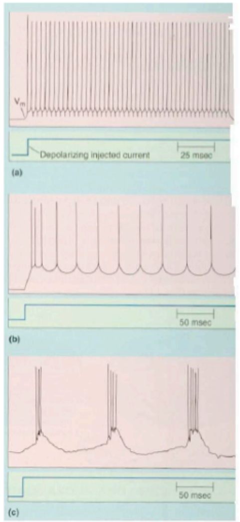


Fig. 1. Electrical circuit representing membrane. $R_{Na} = 1/g_{Na}$; $R_K = 1/g_K$; $R_l = 1/\bar{g}_l$. R_{Na} and R_K vary with time and membrane potential; the other components are constant.

The nonlinear behavior of neuronal signaling

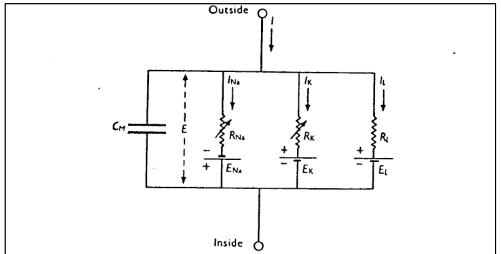
[*Neuroscience*. M. Bear et al.]





A two-port circuit
Classified according to step response

Ion channels regulate ion conductances



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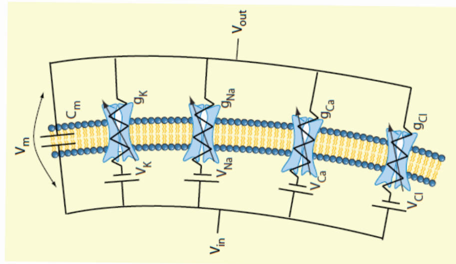

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Hodgkin-Huxley model is a rigorous state-space model

$$
\begin{aligned}
C_m \dot{V}_m = & - \bar{g}_{Na} m_{Na}^3 h_{Na}(V_m - V_{Na}) \\
& - \bar{g}_K m_K^4 (V_m - V_K) \\
& - g_{leak}(V_m - V_{leak}) + I_{app},
\end{aligned}
$$

with

$$
\begin{aligned}
\tau_{m_{Na}}(V_m)\dot{m}_{Na} &= -(m_{Na} - m_{Na,\infty}(V_m)) \\
\tau_{h_{Na}}(V_m)\dot{h}_{Na} &= -(h_{Na} - h_{Na,\infty}(V_m)) \\
\tau_{m_K}(V_m)\dot{m}_K &= -(m_K - m_{K,\infty}(V_m)),
\end{aligned}
$$

where

$$
\begin{aligned}
\tau_X(V_m) &= \frac{1}{\alpha_X(V_m) + \beta_X(V_m)} \\
X_\infty(V_m) &= \frac{\alpha_X(V_m)}{\alpha_X(V_m) + \beta_X(V_m)},
\end{aligned}
$$

and

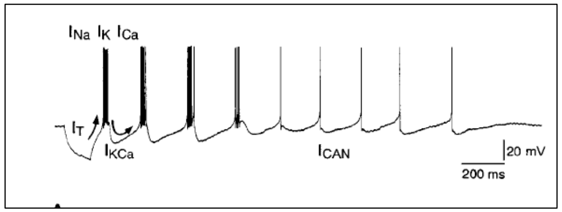
$$
\begin{aligned}
\alpha_{m_{Na}}(V_m) &= \frac{V_m + 35}{10\left[1 - e^{-(V_m+35)/10}\right]} \\
\beta_{m_{Na}}(V_m) &= 4e^{-(V_m+60)/18} \\
\alpha_{h_{Na}}(V_m) &= 0.07e^{-(V_m+60)/20} \\
\beta_{h_{Na}}(V_m) &= \frac{1}{1 + e^{-(V_m+30)/10}} \\
\alpha_{m_K}(V_m) &= \frac{V_m + 50}{100\left[1 - e^{-(V_m+50)/10}\right]} \\
\beta_{m_K}(V_m) &= 0.125e^{-(V_m+60)/80}.
\end{aligned}
$$

The Behavior Is All There Is
(Jan, CSM 2007)

Digging into the electrophysiology of dopamine

Guillaume Drion

Vincent Seutin

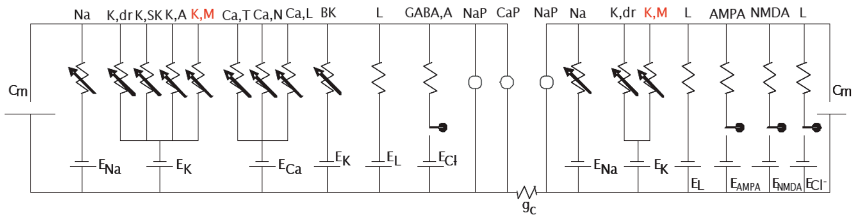


Alessio Franci

What does regulate the electrical behavior of neurons?

Julie Dethier

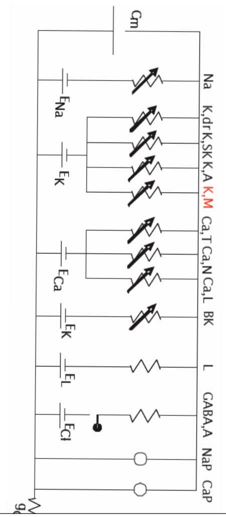
A NL electrical circuit can be complicated...

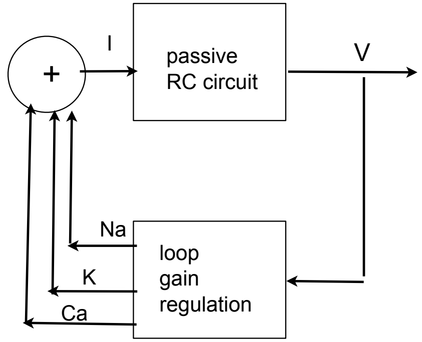


About 130 state variables and 500 parameters
(Canavier et al., 2006; Drion et al. 2010)

The Behavior Is All There Is

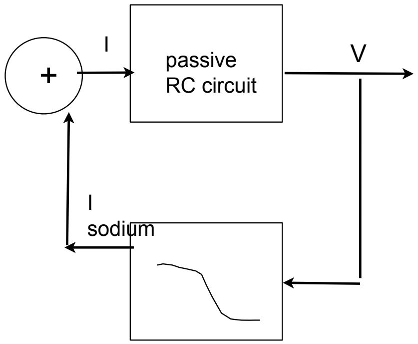
The lesson of a 6 years journey: it is “just” about loop shaping





Why so many loops?

Each loop has the same structure



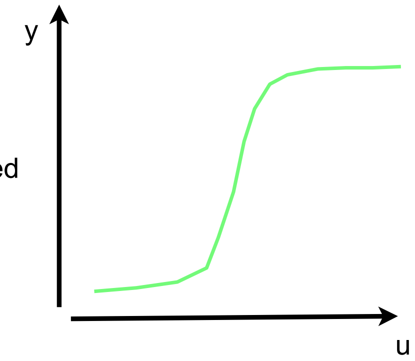
the sodium conductance is modulated by a voltage dependent activation variable:

$$\tau_{m_{Na}}(V_m)\dot{m}_{Na} = (m_{Na} - m_{Na,\infty}(V_m))$$

Three elementary facts about behaviors in neuroscience

Fact one: static behaviors are softly binary

range-limited

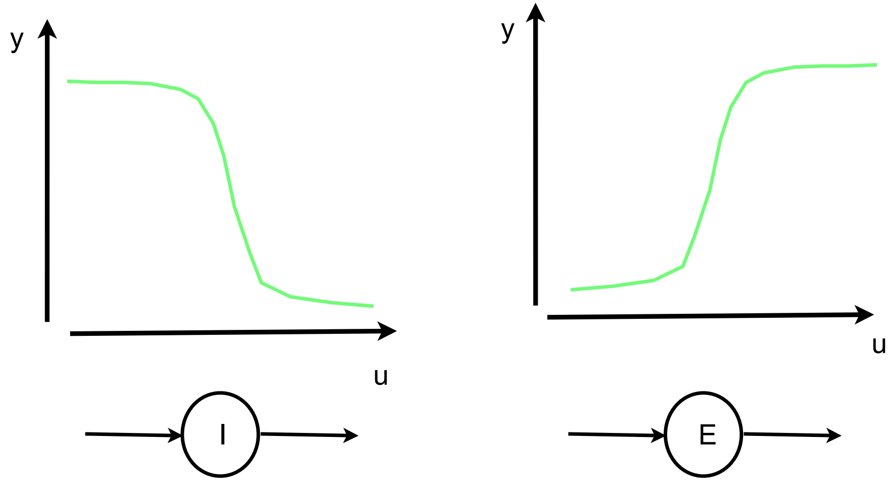


Behavioral keywords:

switches,
(in)activation,
on/off,
ultrasensitive,
...

'Switches' are pervasive across all scales of experimental investigation

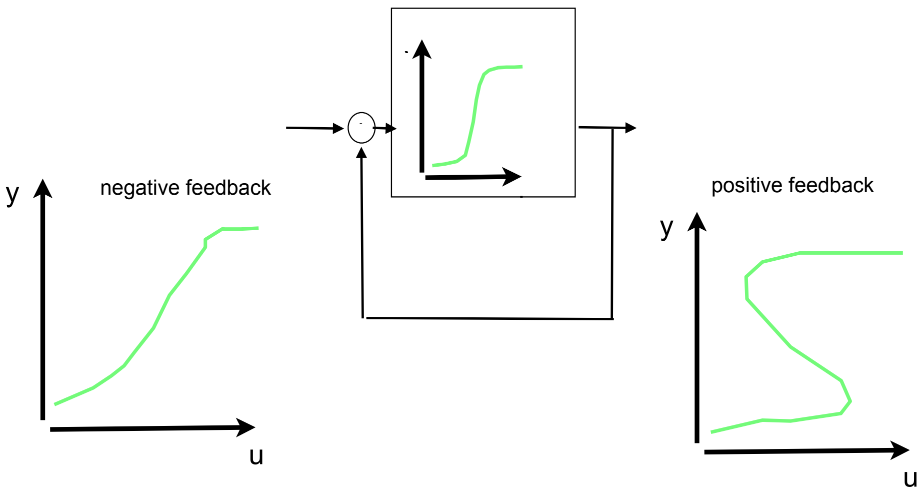
Fact one bis: switches come in two forms:



Inhibition, inactivation

Excitation, activation

Fact two: feedback shapes the switch memory



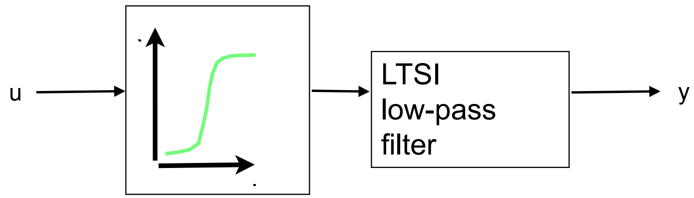
(inhibition, adaptation, ...)

memoryless

(excitation, autocatalysis, ...)

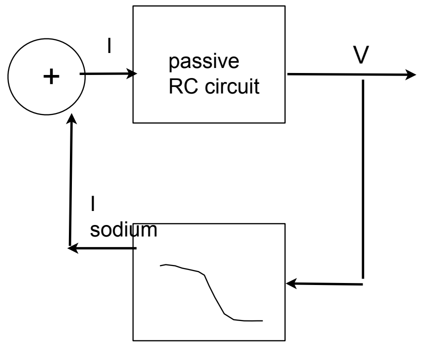
bistability = two-state memory

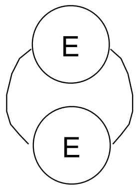
Fact three: natural switches are low-pass

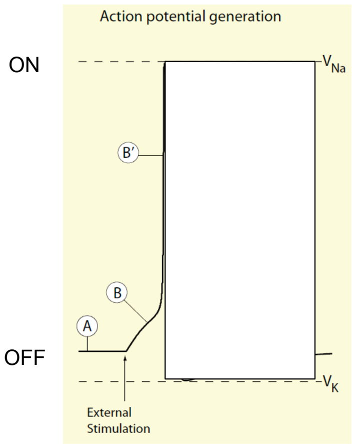


current activation (Hodgkin-Huxley, 1952), neural field model (Amari, 1977), Hopfield model (1984), Turing reaction diffusion model (1952), ...

Sodium activation is a memory switch

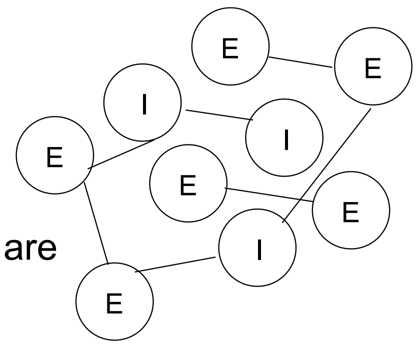






Biological models are about E and I interconnections

Hopfield, Wilson-Cowan, Amari,
Cohen-Grossberg, Tyson, Goldbeter,
Alon, Angeli, Sontag, ...



In general, those behaviors are

- mathematically intractable
- Phenomenological
- confined to one biological scale (genes, proteins, currents, ganglia, brain areas, ...)

Claim: biological behaviors have an additional property: LOCALIZATION

This property makes them

- multiresolution
- mathematically tractable across time and spatial scales

Thinking of a dynamical system as a behavior, and of interconnection as variable sharing, gets the physics right.

(Jan, CSM97, but without the ‘s’ in personal communications...)

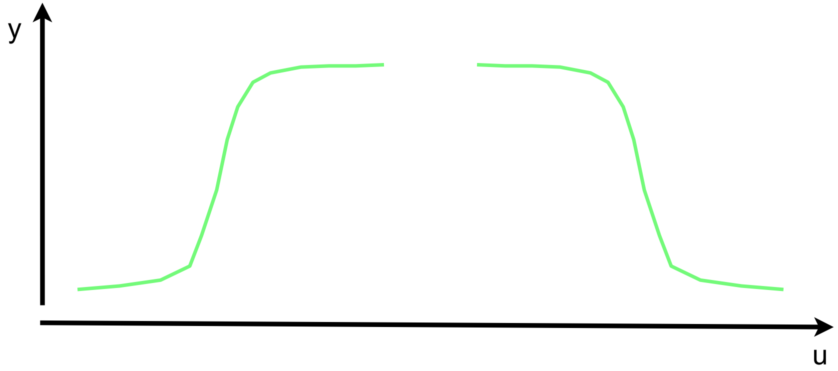
The lesson of multiresolution signal theory

- Wavelets have revolutionized signal processing.
- Localizing a signal in time-frequency and in space-frequency is the key to zooming and tearing, i.e. a multiresolution signal theory.

By an interconnected system, we mean a system that consists of interacting subsystems modeled by tearing, zooming, and linking.

Jan, again.

Localizing a behavior by feedback : the "E then I" loop-shaping principle



E "THEN" I

A pervasive loop-shaping principle

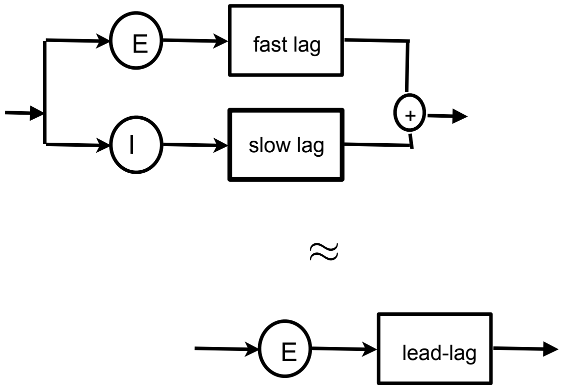
"Gain" E "THEN" I

Static : activation near low state/ inactivation near high state

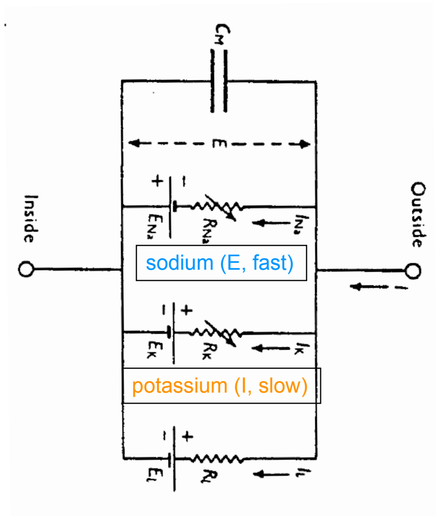
Time-frequency domain: fast activation / slow adaptation

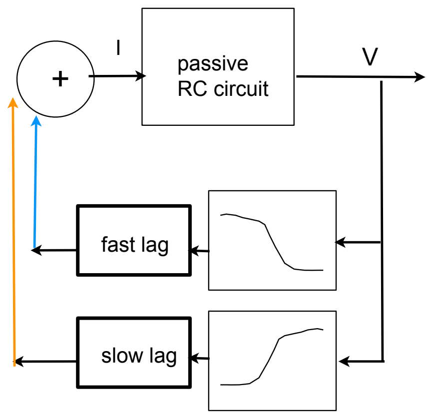
space-frequency domain: short range excitation / long range inhibition

A Bode look at localization in time-frequency domain



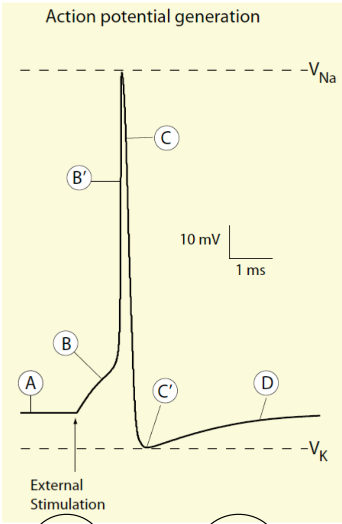
Hodgkin-Huxley electrical circuit is a memory localized in the frequency domain





The behavior of Hodgkin-Huxley model

ON



OFF

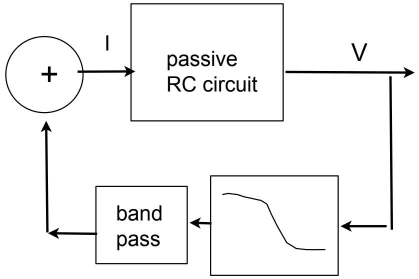
E "THEN" I

E feedback turns memory on

I feedback turns memory off

E then I feedback in time
=
action potential
=
fast and non-lasting two-state memory

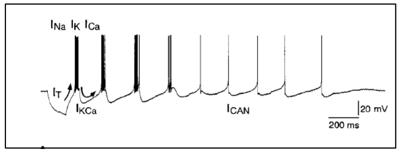
A memory localized in the time-frequency domain

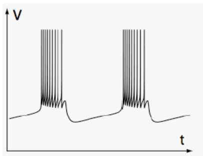


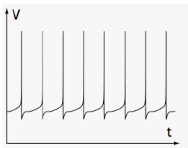
A memory switch with respect to fast signals
A memoryless switch with respect to slow signals

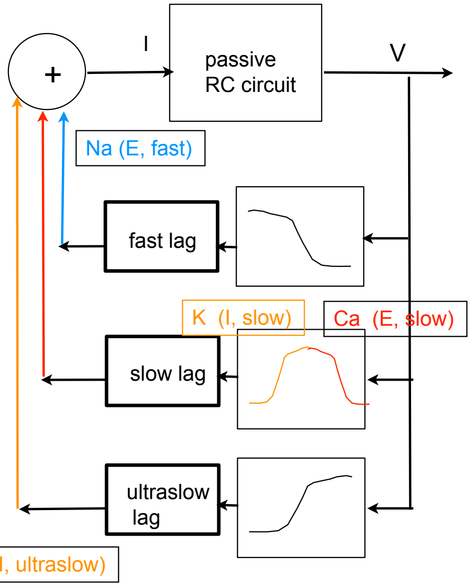
Neuronal realization :
a feedback loop shaped with one fast excitatory current (sodium) and one slow inhibitory current (potassium)

The two-resolution structure of general neurons









Conclusions

- A great deal can be learned about the electrical behavior of neurons using Bode's viewpoint on circuits.
- "E then I" loop-shaping is pervasive in biological models.
- A localized memory is an atom for multiresolution feedback systems.
- It localizes behaviors in time, space, and range. This opens the path to a tractable theory of multi-resolution interconnections.
- A framework suggested by conductance-based models but which suggests generality.

Nonlinear control theorists have a great deal to learn from Keith and Jan !

Keith Glover, Jan Willems

Articles (include patents) Legal documents

Stand on the shoulders of giants